Illinois Institute of Technology

Physics

M.Sc. Comprehensive and Ph.D. Qualifying Examination

PART II

Saturday, January 13, 2018

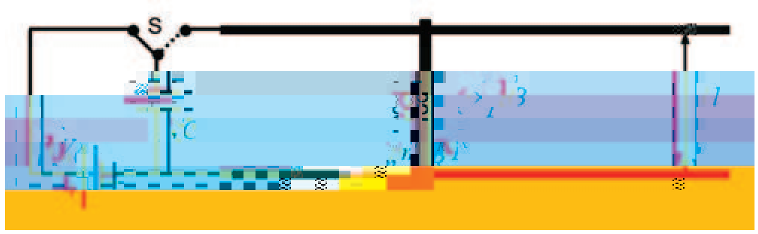
12:00 - 4:00 PM

General Instructions

Physical Constants

Speed of light in vacuum	c	=	$2{:}998 \quad 10^{8}$ m/s
Planck's constant	h	=	$6{:}626 \quad 10^{\;34}$ J s
	$\hbar$	=	$h{=}2$
		=	$1{:}055 \quad 10^{\;34}$ J s
		=	$6{:}582 \quad 10^{\;16}$ eV s
Permeability constant	$_{o}$	=	$4 \quad 10^{\;7}$ N/A^{2}
Permittivity constant	$\frac{1}{4\;\;_{o}}$	=	$8{:}988 \quad 10^{9}$ N m^{2}/C^{2}
Fine structure constant		=	$\frac{e^{2}}{4\;\;_{o}\hbar c}$
		=	$7{:}30 \quad 10^{\;3} = \frac{1}{137}$
Gravitational constant	G	=	$6{:}67 \quad 10^{\;11}$ m^{3}/s^{2} kg
Avogadro's number	N_{A}	=	$6{:}023 \quad 10^{23}$ mole$^{\;1}$
Boltzmann's constant	k	=	$1{:}381 \quad 10^{\;23}$ J/K
		=	$8{:}617 \quad 10^{\;5}$ eV/K
kT at room temperature	k 300 K	=	0.0258 eV
Universal gas constant	R	=	$8{:}314$ J/mole K
Stefan-Boltzmann constant		=	$5{:}67 \quad 10^{\;8}$ W/m^{2} K^{4}
Electron charge magnitude	e	=	$1{:}$

Problem 1:



One end of a horizontal track of width l and negligible resistance is connected to a capacitor of capacitance C charged to voltage V_0 of polarity shown in the gure. The inductance of the assembly is negligible. The system is placed in a homogeneous vertical magnetic eld B pointing into the page. A frictionless conducting rod of mass m and resistance R is placed perpendicular onto the track. After the capacitor is fully charged the position of the switch S is changed from the position indicated by the full line to the position indicated by the dotted line, and the rod starts moving.

(a) In which direction does the rod move, and why?

(b) What is the maximum velocity that the rod acquires?

Problem 2:

A charge density $_0$ is placed at time $t = 0$ in a small region in the interior of a homogeneous charge-neutral material that has electrical conductivity .

(a) Derive an expression for the time evolution of the charge density in that region, $_c(t)$, with $_c(0) = {}_0$. Hint: use a continuity equation.

(b) Estimate how long it will take (in seconds) for the charge density to decrease to 1/1000 of its initial value if the material is (i) copper with conductivity = 1=(2 10 8 m) and (ii) quartz with conductivity = 1=(10^{16} m).

Use $''_0 = 8{:}85 \quad 10^{12}$ C^2/Nm^2.

Problem 3:

A surface of an in nite cylinder of radius R is charged with the charge density ( ) =

Problem 4:

A one-dimensional particle of mass m and energy E is incident on the δ-function potential $V(x) = V_0\,\delta(x)$.

(a) Find the reflection and transmission coefficients.

(b) Find the phase shift ϕ of the transmitted wave, and the difference

$$\phi(E \to \infty) - \phi(E \to 0).$$

Problem 5:

Consider an electron constrained to move in the xy plane under the influence of a uniform magnetic field of magnitude B oriented in the $+\hat{z}$ direction. The Hamiltonian for this electron is

$$\mathrm{H} = \frac{1}{2m}\left[\left(\mathrm{p}_x - \frac{e}{c}A_x\right)^2 + \left(\mathrm{p}_y - \frac{e}{c}A_y\right)^2\right],$$

where m and e are the mass and charge of the electron, and c is the speed of light.

(a) Find a suitable expression for $\vec{A}$ so that p_x is a constant of motion for the above Hamiltonian.

(b) With this choice for $\vec{A}$, show that the eigenfunctions of H can be written in the form

$$\psi(x, y) = e^{\frac{i}{\hbar}p_x x}\,\phi(y),$$

where $\phi(y)$ satisfies the Schrödinger equation for a one-dimensional harmonic oscillator whose equilibrium position is $y = y_0$. Find the effective spring constant k for this oscillator and the shift of the origin y_0 in terms of p_x, B, m, e, c.

(c) Find the energy eigenvalues for this system, and indicate degeneracies.

(d) For the remainder of the problem, suppose we further restrict the particles to live in a square of side length L. Suppose we demand periodic boundary conditions. What are the possible values of p_x?

Problem 6:

Prove the following relation, where $\hat{\vec{L}}$ is an angular momentum operator:

$$\left[\hat{\vec{L}}, p_x^2 + p_y^2 + p_z^2\right] = 0.$$

Problem 7:

In a December IIT M.S. Thesis, the production and decay of a supersymmetric single top *squark* $\tilde{t}$