

# MATH 557 Spring 2010 – Probabilistic Methods in Combinatorics

**Course Description:** Graduate level introduction to probabilistic methods, including linearity of expectation, the deletion method, the second moment method and the Lovász Local Lemma. Many examples from classical results and recent research in combinatorics and graph theory will be included throughout, including from Ramsey Theory, random graphs, coding theory, and number theory. (3-0-3)

**Enrollment:** Graduate elective

**Textbook:** Alon and Spencer, *The Probabilistic Method*, 3<sup>rd</sup> ed., Wiley-Interscience, 2008, ISBN: 0470170204.

**Other required material:**

**Prerequisites:** Graduate status or consent of the instructor

## Objectives:

1. Students will master the definitions and basic theorems of discrete probability spaces necessary for understanding and applying probabilistic methods.
2. Students will achieve a firm command of the theory behind the basic probabilistic methods: linearity of expectation, the deletion method, the second moment method, and the Lovász Local Lemma.
3. Students will explore the basic probabilistic methods by application to important classical examples in combinatorics, graph theory, coding theory, computational geometry and number theory.
4. Students will apply probabilistic methods to solve important problems in recent probabilistic methods research.
5. Students will do a project in which they apply probabilistic methods to a problem of their own choosing

Discrete Probability Spaces	4
a. Basic definitions and properties: subadditivity, conditional probability, pairwise and mutual independence	
b. Expected value: linearity of expectation and conditional expectation	
c. Edge exposure and the method of deferred decisions	
2. Initial applications	5
a. Diagonal Ramsey numbers, tournaments, 2-coloring of hypergraphs	
b. Set systems, sum-free subsets, and the Erdős-Ko-Rado Theorem	
3. Linearity of Expectation and the First Moment Method	9
a. Basics: Markov's Inequality and Linearity of Expectation; derandomization via conditional expectation	
b. Applications: tournaments, bipartite subgraphs and crossing sets, balancing vectors, Rényi-Ulam liar games	

4.	Alterations of the First Moment Method	8
	a. The deletion method	
	b. Improving applications: Ramsey numbers, hypergraph recoloring	
	c. Applications: independent sets, geometric packing, covering codes, high girth and high chromatic number, and the Guessing Secrets problem	
5.	The Second Moment Method	8
	a. Variance and covariance, concentration, Chebyshev's Inequality and Chernoff bounds	
	b. Applications: density of primes, random graphs and threshold functions, clique number, distinct sums, Hamiltonian paths, random geometric graphs	
6.	The Lovász Local Lemma	6
	a. The Lemma: basic and symmetric cases	
	b. Applications: hypergraph coloring, Ramsey numbers, geometric packing, independent sets, and Latin Transversals	
	c. Algorithmic aspects and derandomization	
7.		