Illinois Institute of Technology

Physics

M.Sc. Comprehensive and Ph.D. Qualifying Examination

PART I

Thursday, August 22, 2019

4:00{8:00 PM

General Instructions

1. Each problem is to be done on a separate booklet. Label the front of each book with the identifying code letter you picked, the part number of the exam, and the number of the problem only; for example: A-I.6. Do not write your name or IIT ID number on any material handed in for grading.
2. Any numerical data not speci ed in a problem should be found in the table of constants at the front of the exam.
3. *DON'T PANIC:* It is not expected that each student will completely solve every problem. However, it is advisable to do a thorough job on those problems that you do solve.

Physical Constants

Speed of light in vacuum	c	=	$2{:}998 \quad 10^{8}$ m/s
Planck's constant	h	=	$6{:}626 \quad 10^{\ 34}$ J s
	~	=	h=2
		=	$1{:}055 \quad 10^{\ 34}$ J s
		=	$6{:}582 \quad 10^{\ 16}$ eV s
Permeability constant	$_{o}$	=	$4 \quad 10^{\ 7}$ N/A^{2}
Permittivity constant	$\frac{1}{4 \ _{o}}$	=	$8{:}988 \quad 10^{9}$ N m^{2}/C^{2}
Fine structure constant		=	$\frac{e^{2}}{4 \ _{o}\tilde{}c}$
		=	$7{:}30 \quad 10^{\ 3} = \frac{1}{137}$
Gravitational constant	G	=	$6{:}67 \quad 10^{\ 11}$ m^{3}/s^{2} kg
Avogadro's number	N_{A}	=	$6{:}023 \quad 10^{23}$ mole ^{1}
Boltzmann's constant	k	=	$1{:}381 \quad 10^{\ 23}$ J/K
		=	$8{:}617 \quad 10^{\ 5}$ eV/K
kT at room temperature	k 300 K	=	0.0258 eV
Universal gas constant	R	=	$8{:}314$ J/mole K
Stefan-Boltzmann constant		=	$5{:}67 \quad 10^{\ 8}$ W/m^{2} K^{4}
Electron charge magnitude	e	=	$1{:}602 \quad 10^{\ 19}$ C
Electron rest mass	m_{e}	=	$9{:}109 \quad 10^{\ 31}$ kg
		=	0.5110 MeV/c^{2}
Neutron rest mass	m_{n}	=	$1{:}675 \quad 10^{\ 27}$ kg
		=	939.6 MeV/c^{2}
Proton rest mass	m_{p}	=	$1{:}672 \quad 10^{\ 27}$ kg
		=	938.3 MeV/c^{2}
Deuteron rest mass	m_{d}	=	$3{:}343 \quad 10^{\ 27}$ kg
		=	1875.6 MeV/c^{2}
Atomic mass unit (C^{12} = 12)	u	=	$1{:}661 \quad 10^{\ 27}$ kg
		=	931.5 MeV/c^{2}
Mass of earth	M_{E}	=	$5{:}98 \quad 10^{24}$ kg
Radius of earth	R_{E}	=	$6{:}37 \quad 10^{6}$ m
Mass of sun	M_{S}	=	$1{:}99 \quad 10^{30}$ kg
Radius of sun	R_{S}	=	$6{:}96 \quad 10^{8}$ m
Gravitational acceleration at earth's surface	g	=	9.81 m/s^{2}
Atmospheric pressure		=	$1{:}01 \quad 10^{5}$ N/m^{2}
Radius of earth's orbit		=	$1{:}50 \quad 10^{11}$ m
Radius of moon's orbit		=	$3{:}84 \quad 10^{8}$ m

Conversion Factors

1 eV	=	$1{:}602 \quad 10^{\ 19}$ J	1 J	=	$6{:}242 \quad 10^{18}$ eV
1 A	=	$10^{\ 10}$ m	1 Fermi	=	$10^{\ 15}$ m
1 barn (b)	=	$10^{\ 28}$ m^{2}	1 in	=	2.54 cm
0 Celsius	=	273.16 K	1 cal	=	4.19 J

Problem 4: A satellite travels in a circular orbit of radius r_0. Its rocket motor fires, suddenly increasing its velocity by 8% along its direction of motion. What is the apogee of the new orbit? Make a sketch superimposing the new orbit on the original orbit.

Problem 5: Consider a three-dimensional ideal gas placed into a spherically symmetric potential, given by the formula:

$$V(r) = \begin{cases} \infty; & r \leq R \\ U_0 \ln(r/R); & r > R \end{cases}$$

(a) Find a single particle partition function.

A useful Gaussian integral:

$$\int_0^{\infty} x^2 e^{-x^2/a} dx = \sqrt{\pi a^3}/4$$

(b) Find an N-particle partition function for $N \gg 1$. A useful Stirling formula:

$$N! \approx \left(\frac{N}{e}\right)^N$$

in this case.

(c) What is the highest possible gas temperature?

(d) Find a (Helmholtz) free energy of the gas.

(e) Find an internal energy of the gas.

(f) Find a specific heat (capacity) of the gas.

Problem 6: Calculate a chemical potential for an ideal **two-dimensional** non-relativistic Fermi-gas of a surface density n_S and arbitrary temperature T. A fermion mass is equal to m and spin is $1/2$. Then find the chemical potential for a high and low temperature limits.

Problem 7: Using statistical mechanical principles, estimate the mean thickness of the earth's atmosphere in terms of the mass m of a nitrogen molecule, the gravitational acceleration g, and the average atmospheric temperature T. Evaluate your result numerically.

Problem 8:

(a) A cylindrical glass optical fiber (radius R) with index of refraction, n, propagates light by total internal reflection at the air-glass interface. Give an expression for the maximum angle of incidence, , at the flat end of the fiber22.928 Td [(e)]TJnrnal reflectioblem8