Illinois Institute of Technology

Physics

M.Sc. Comprehensive and Ph.D. Qualifying Examination

PART II

Saturday, August 24, 2019

1:00{5:00 PM

General Instructions

1. Each problem is to be done on a separate booklet. Label the front of each book with the identifying code letter you picked, the part number of the exam, and the number of the problem only; for example: A-I.6. Do not write your name or IIT ID number on any material handed in for grading.
2. Any numerical data not speci ed in a problem should be found in the table of constants at the front of the exam.
3. *DON'T PANIC:* It is not expected that each student will completely solve every problem. However, it is advisable to do a thorough job on those problems that you do solve.

Physical Constants

Speed of light in vacuum	c	=	2:998 10^{8} m/s
Planck's constant	h	=	6:626 10^{-34} J s
	~	=	h=2
		=	1:055 10^{-34} J s
		=	6:582 10^{-16} eV s
Permeability constant	o	=	4 10^{-7} N/A^{2}
Permittivity constant	$\frac{1}{4\ \ o}$	=	8:988 10^{9} N m^{2}/C^{2}
Fine structure constant		=	$\frac{e^2}{4\ \ o \sim c}$
		=	7:30 10^{-3} = $\frac{1}{137}$
Gravitational constant	G	=	6:67 10^{-11} m^{3}/s^{2} kg
Avogadro's number	N_A	=	6:023 10^{23} mole^{-1}
Boltzmann's constant	k	=	1:381 10^{-23} J/K
		=	8:617 10^{-5} eV/K
kT at room temperature	k 300 K	=	0.0258 eV
Universal gas constant	R	=	8:314 J/mole K
Stefan-Boltzmann constant		=	5:67 10^{-8} W/m^{2} K^{4}
Electron charge magnitude	e	=	1:602 10^{-19} C
Electron rest mass	m_e	=	9:109 10^{-31} kg
		=	0.5110 MeV/c^{2}
Neutron rest mass	m_n	=	1:675 10^{-27} kg
		=	939.6 MeV/c^{2}
Proton rest mass	m_p	=	1:672 10^{-27} kg
		=	938.3 MeV/c^{2}
Deuteron rest mass	m_d	=	3:343 10^{-27} kg
		=	1875.6 MeV/c^{2}
Atomic mass unit (C^{12} = 12)	u	=	1:661 10^{-27} kg
		=	931.5 MeV/c^{2}
Mass of earth	M_{E}	=	5:98 10^{24} kg
Radius of earth	R_{E}	=	6:37 10^{6} m
Mass of sun	M_{S}	=	1:99 10^{30} kg
Radius of sun	R_{S}	=	6:96 10^{8} m
Gravitational acceleration at earth's surface	g	=	9.81 m/s^{2}
Atmospheric pressure		=	1:01 10^{5} N/m^{2}
Radius of earth's orbit		=	1:50 10^{11} m
Radius of moon's orbit		=	3:84 10^{8} m

Conversion Factors

1 eV	=	1:602 10^{-19} J	1 J	=	6:242 10^{18} eV
1 A	=	10^{-10} m	1 Fermi	=	10^{-15} m
1 barn (b)	=	10^{-28} m^{2}	1 in	=	2.54 cm
0° Celsius	=	273.16 K	1 cal	=	4.19 J

Problem 1: A long straight piece of copper (Cu) wire with a cross sectional radius a, and electrical resistance R, is bent in half to form two long parallel wires of length l whose centers are a distance d apart.

(a) Neglecting any flux within the wire itself, find the self inductance L of the wire in this configuration.

(b) Assume a battery of emf $\mathcal{E}$ is connected to the wire at time $t = 0$. Derive an expression for the current $i(t)$.

Problem 2: A point charge q is located at distances a and b from two perpendicular conducting half-planes, both at zero potential. Calculate the force acting on the charge q.

Problem 3: A surface of a non-conducting infinitely thin spherical shell of radius R is charged with the surface charge density $\sigma(\theta) = \sigma_0 \cos\theta$. Find the electrostatic potential inside and outside the shell.

Hint: Recall the Laplace operator in spherical system of coordinates:

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

Problem 6: Using WKB (quasi-classical) approximation, estimate a transmission coefficient through a potential barrier:

$$U(x) = \begin{cases} 0 & x > |x_0| \\ U_0 - \frac{1}{2} m \omega^2 x^2 & x \le |x_0| \end{cases}$$

for a particle of mass m and energy $0 < E < U_0$. To define x_0, use a continuity requirement for the potential energy $U(x)$.

Problem 7: